These contributions are important steps forward in the use of advance decision theoretical framework in the context of climate change, and create an important background for the specific implementation of uncertainty in the IAMs participating in ADVANCE, which is currently ongoing and which will be documented in the deliverable 4.3.

Selection of climate policies under the uncertainties outlined in IPCC AR5

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Strategies for dealing with climate change must account for all the relevant uncertainties and manage the resulting risks[1,2]. Here we draw on the fifth assessment report of the Intergovernmental Panel on Climate Change (IPCC AR5)[3–5] to infer probability distributions of mitigation costs, temperature changes and resulting climate change impacts, thus using the best available knowledge to quantify the deep uncertainties associated with climate policies. We have selected a range of climate policies according to the different decision-making criteria as regard uncertainty, risk aversion and intertemporal discounting. Our findings show that the choice of the decision-making criteria is as important as that of the thoroughly analysed time discount factor. Climate policies consistent with 2°C are compatible with some criteria and some specifications, but not with the standard expected utility framework.

Decision-making in the context of climate change has to rely on a risk management approach, which accounts for the different sources of uncertainty, the presence of non-linearities in the climate system, as well as the potentially different risk attitudes of stakeholders[2]. Including uncertainty in the decision framework requires its quantification, and increases the complexity of the assessment exercise. Therefore, it is important to use the best available knowledge in a transparent framework which allows individuals, stakeholders and policymakers to visualise the key uncertainties and translate them into alternative climate policies based on their preferences.

The role of preferences is important for several reasons. Intertemporal preferences have long been recognized as key factors, given the long-lasting nature of the climate problem[6,7]. Making climate policy also implies deep uncertainty, since many of the uncertainties surrounding climate change are difficult to quantify. In addition, while mitigation costs and related uncertainty concern primarily events that are commensurable with monetary losses, uncertainty about climate impacts might incorporate events that cannot often be assessed in monetary terms. As a result, the standard expected utility[8] and subjective expected utility[9] frameworks are not always suitable for making decisions in this context. Alternative decision-making criteria, for example in accounting for ambiguity aversion — the tendency to choose an option with fewer unknown elements over one with many unknown elements — should be included as plausible alternative preferences when decisions are being made about climate policy[10,11]. Yet, few quantitative assessments of the role of preferences for uncertainties have been performed so far in the context of climate change.

The literature on the role of uncertainty in climate policy decision-making has mostly relied on either analytical or simplified, numerically calibrated, integrated assessment models (IAMs) such as DICE[12]. In such contexts different decision-making criteria and preferences over risks have been shown to have a significant impact on the optimal abatement strategy[13–18]. However, these exercises lack detail on the representation of the mitigation options and costs, since they rely on simplistic climate models and on a single economic model. Large-scale energy-economy-climate models, which capture the main interrelationships between economic, energy and climate systems and are key providers of input to the IPCC AR5, have incorporated uncertainty only partially due to computational limitations[19]. Therefore, uncertainty is mostly treated by performing Monte-Carlo simulations, where several parameters vary together and are chosen randomly from a discrete or continuous predefined distribution[20–22]. Recently[23] a distribution of climate change mitigation costs were computed by combining the cost estimates issued from many scenarios with probabilistic projection of temperature increase, but they rely on a single economic model and do not explore different decision-making criteria. Our work is more in line with the policy risk assessment framework[24]. This approach is based on the generation of multiple possible futures, spanning a wide range of conditions which are evaluated and ranked according to several criteria.

In this paper we propose a method for selecting climate policies that implies deep uncertainty and accounts for different risk and time preferences. In order to do so, we quantify uncertainty by building probability density functions of mitigation costs, temperature increase and climate change impacts for various alternative climate policies, using the best available knowledge reviewed in the most recent assessments of the three working groups of IPCC's AR5. The decision variable or climate policy indicator, in our framework, is the carbon budget, that is the cumulative CO 2 emissions over the 21st century (2010–2100). Carbon budgets are robust policy indicators since they are strictly related to global warming[25–27]and climate targets[27] and thus climate change impacts, as well as abatement costs.

Figure 1 illustrates our approach. We extract emission projections and associated mitigation costs from the AR5 WGIII Scenario database[5], which includes the outcomes from many long-term scenarios produced by the most well-established integrated assessment models (IAMs). The database spans a wide range of policy stringency, and thus of associated carbon budgets, covering the whole range of the representative concentration pathways (RCP)[28]. The variety of models included allows us to capture different assumptions concerning technologies and their evolution, integration of energy, economy and the land use sectors, but also more structural differences such as the sectoral and geographical disaggregation or solution concept. Such heterogeneity spans a wide set of socio-economic futures and mitigation opportunities, most probably wider than would be possible by simply changing assumptions within a single model. The probabilistic relation between mitigation costs, harmonized across different metrics, and carbon budgets is found to be non-linear, convex and highly uncertain (see Supplementary Figure S1). Furthermore, the uncertainty of mitigation costs is increasing with time (see Supplementary Figure S2).

For each IAM simulation we generate probabilistic temperature projections, given the carbon dioxide emissions and the radiative forcing pathways to it associated in the database. These are obtained from a probabilistic climate model[29], calibrated to emulate the temperature projections from the fifth phase of the coupled model intercomparison project[30](see Supplementary Figure S3). Hence, we include in our analysis climate model uncertainty, which is known to significantly increase the impact estimates, and so far has been very rarely accounted for[31].

Finally, we link the temperature increase to global economic impacts as reported in the AR5

WGII[4]. These estimates do not include all the potential damage from climate change[32,33], but represent the best currently available knowledge, and have been used for calculating the social cost of carbon in several analyses[34]. However, given the weak consensus on the functional form relationship between temperature increase and the economic impacts of climate change and the few estimates available, especially for temperatures above 3°C[35], we consider three alternative impact function specifications. In addition to the commonly used quadratic specification, we also consider the exponential and sextic impact functions. When provided, we integrate the uncertainty surrounding the impact estimates in the impact function calibration. As shown in Supplementary Figure S4, all three functional forms are consistent with the data, but they exhibit drastically different behaviour at high temperatures.

Each carbon budget is then related to three time-dependent distributions of payoffs, computed by combining mitigation costs and climate change damage distributions for each of the three functional forms, conditional on the temperature realization. The results are shown in Figure 2for three illustrative carbon budgets (the payoffs are shown in Supplementary Figure S5). In case of low carbon budgets (left panel), climate damages are kept under reasonable control and the choice of the damage functional form has a negligible effect; rather it is mitigation costs that dominate. These costs increase in time with the rate of emission reduction. So does the related uncertainty. As we move to higher budgets (central and right-hand panel), the balance shifts and the importance of climate change damages and their uncertainty tend to prevail. In both cases, the difference between the three climate change damage functions becomes more marked, especially late into the century when high temperature realizations are more likely to occur. The chart indicates that even for the larger budgets, the quadratic functional specification fails to capture significant fat-tailed impact events, whereas this is not the case for the exponential and sextic functions. It is worth noting that IAM scenarios without explicit climate policies (normally referred to as BAU scenarios) entail carbon budgets which normally well exceed the highest budget shown in Figure 2. This confirms that the quadratic form, although widely employed in the literature[12], might not be appropriate for assessing the welfare impacts of high temperature changes[36]. As mitigation efforts decrease, so do mitigation costs and the associated uncertainty which is almost negligible in the right-most panel.

We consider three decision-making frameworks, two probabilistic and one non-probabilistic, in order to represent different attitudes of the decision maker. The criteria are built upon the flexible classical subjective expected utility framework[37], and are outlined in Supplementary Table S1. For the probabilistic frameworks, we contrast the standard subjective expected utility (SEU) to a maxmin expected utility (maxmin EU). The former takes expectations over states of nature and over the three alternative damage functional forms (each considered equally likely for lack of better information). The latter, instead, combines the expected utility approach with the maximin criterion[38]. Indeed, it selects the carbon budget on the basis of the expected damage of the most pessimistic of three climate damage functional forms. The two frameworks incorporate, respectively, neutrality and full aversion to the ambiguity related to the specification of the damage function. For the non-probabilistic framework, we pick the maxmin criterion[38], which focuses on the set of worst-case outcomes associated to each carbon budget. Additional frameworks have been proposed, but these are among the best known and provide useful limiting cases[11].

The selected ranges of carbon budgets for different decision criteria, time preference and risk aversion are reported in Figure 3. The figure shows the relationship between time preference and climate policy, namely higher discounting leading to higher CO_2 budgets. It also allows quantifying the role of the decision-making rule on policy. The most prudent criterion — maxmin — is associated with the lowest carbon budgets. Probabilistic rules based on expectations lead to higher carbon budgets, especially when aversion to ambiguity is not accounted for, as with subjective expected utility. On the other hand, risk aversion does not have a significant unilateral effect, since we are accounting for uncertainties on both mitigation costs and impacts, whose uncertainties dominate for low and high budgets respectively. This result is in line with some recent literature[18].

The impact of decision rules is shown to be as large as or larger than that of time discounting. The SEU framework prescribes budgets which are at least 1000 GtCO 2 higher than in the case of maxmin. The increase is roughly attributable equally to ambiguity aversion about the damage function (the gap between SEU and maxmin EU), and to taking expectations versus focusing on worst realizations (the gap between maxmin EU and maxmin). Our results also indicate that carbon budgets compatible with a radiative forcing of 2.6W/m 2 — which is associated with the 2°C target — can be optimal for several specifications; however, this appears never to be the case for the expected utility framework, which is also the one employed in the vast majority of cost-benefit analyses of climate change.

This paper uses the vast amount of data and information collected in the IPCC AR5 to quantify key uncertainties associated to climate change and implements a methodology for selecting climate policies under different preference structures. Our results show that aversion to deep uncertainty has a major impact on the selection of policies, as a result of the scarcity of knowledge that still characterizes the literature on climate change impacts[39]. This clearly points to the need for additional research in trying to understand and better quantify a wider set of impacts as well as to cover a larger spectrum of potential temperature increases. Similarly, mitigation cost estimates are still very uncertain, and in many instances fail to include important economic feedbacks as well as ancillary benefits. Yet, although learning might partly reduce uncertainty, it is clear that several key sources of uncertainty are likely to remain vast in the years to come. So far, uncertainty has exacerbated the polarization in the public debate over climate change policies. On one hand, it has been interpreted as a reason for postponing any action at all, while on the other it has been used as a key argument in favor of stringent mitigation. This analysis reframes this issue in terms of risk management, thus creating a bridge between the more precautious scientific approach and the efficient-based cost-benefit one. By helping decision-makers and the public to see how their preferences with respect to deep uncertainty, risk and time might translate into climate policies, frameworks such as the one presented here can be conducive to achieving a higher political consensus.

Methods

The carbon selection framework described in Figure 1 is based on three subsequent phases of uncertainty quantification, followed by the criteria definition and, lastly, the selection phase. As far as the first phase is concerned, i.e. mitigation cost distribution estimation, we avail of the results of

6 major model inter-comparison projects (AME[40], AMPERE[41], EMF22[42], EMF27[43], LIMITS[44], ROSE[45]), generated by 10 different economic models and accounting for a total of 750 model simulations from the AR5 scenario database (described in Annex II: Metrics and Methodology of AR5 WGIII[5]). The paper focus on 'first best' policies, that is model simulations where climate targets are implemented in the most efficient way (a subset of 250 scenarios), but the results including also 'second best' policies with delay or/and variations in technology availability can be found in the Supplementary Information SI2. From the dataset, we estimate the probabilistic relations between mitigation costs and carbon budgets by means of least square fits. Each carbon budget is actually associated in the original dataset with an emission profile. Carbon dioxide emissions are taken as they are while for non-CO_2 gases we compute the radiative forcing profile. In the second stage, this information is fed into SNEASY[29], a reduced complexity climate model, composed by the simple climate model DOECLIM[46] and a carbon cycle model including feedbacks from the atmospheric CO2 concentration and temperature[47]. To emulate the CMIP5 temperature projections, the SNEASY key parameters are estimated using a Bayesian inversion technique based on the Markov Chain Monte Carlo algorithm, similarly as in recent works[23,27,48]. In the third phase we use the AR5 WGII literature review of 20 estimates of global economic effects of climate change[4] to fit three damage functions, *a quadratic*, *exponential* and *sextic* specifications. By combining mitigation and damage cost distributions, we generate the time-dependent distribution of relative losses associated to each carbon budget. We impose them on the gross world production projection produced by the Organisation for Economic Co-operation and Development (OECD) for the second Shared Socio-economic Pathway (SSP2)[28]. In order to aggregate across states of nature and time

we consider a specification of social preferences that allows to disentangle risk from intertertemporal substitution attitudes, so that we can independently explore the implications of both [49], see Supplementary Information SI1.7 for the detailed formula. We finally apply a set of decision criteria (maxmin, maxmin expected utility and subjective expected utility) to select carbon budgets, by also varying the discount rate and the relative risk aversion parameter values.

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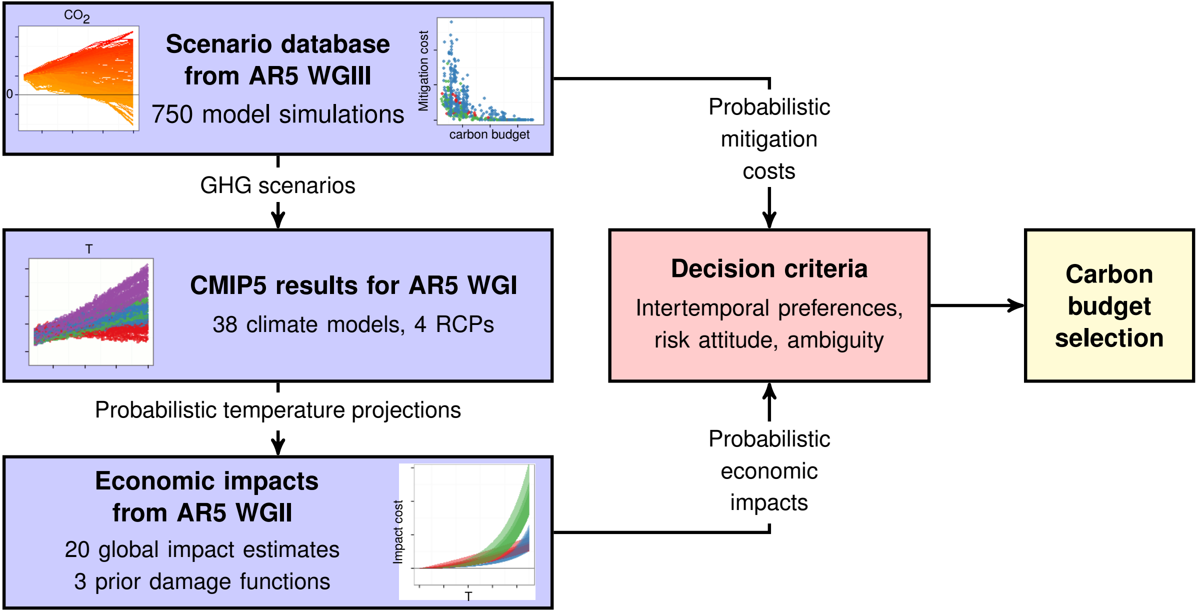
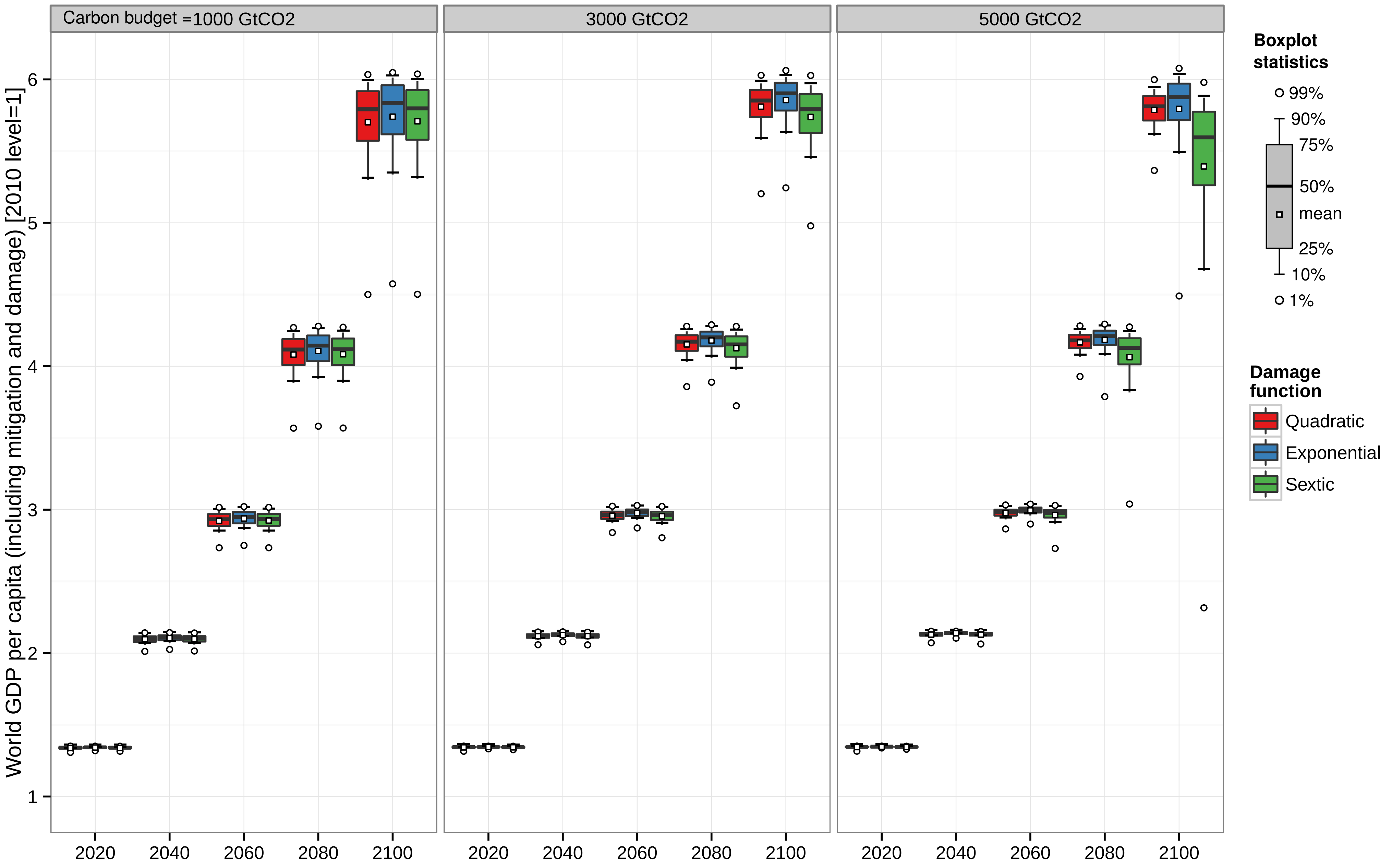
Figure 1 Diagram illustrating the methodology. Greenhouse gas emission scenarios and mitigation costs are extracted from the fifth assessment report, third working group (AR5 WGIII) scenario dataset. Temperature projections are computed from the (CMIP5, AR5 WGI) outcomes. Global economic impacts are generated on the basis of the impact reviews proposed in the fifth assessment report, second working group (AR5 WGII). Finally, carbon budgets are selected from a set of decision rules and preferences. For more details, refer to Supplementary Information S1.

Figure 2 Influence of carbon budgets and damage function on the distribution of the world GDP per capita over the century. The world GDP per capita includes mitigation costs and impacts from climate change, expressed relatively to 2010 levels. Results are provided for three illustrative CO_2 carbon budgets (cumulative emissions over the period 2010–2100) 1000, 3000 and 5000 $GtCO_2$.

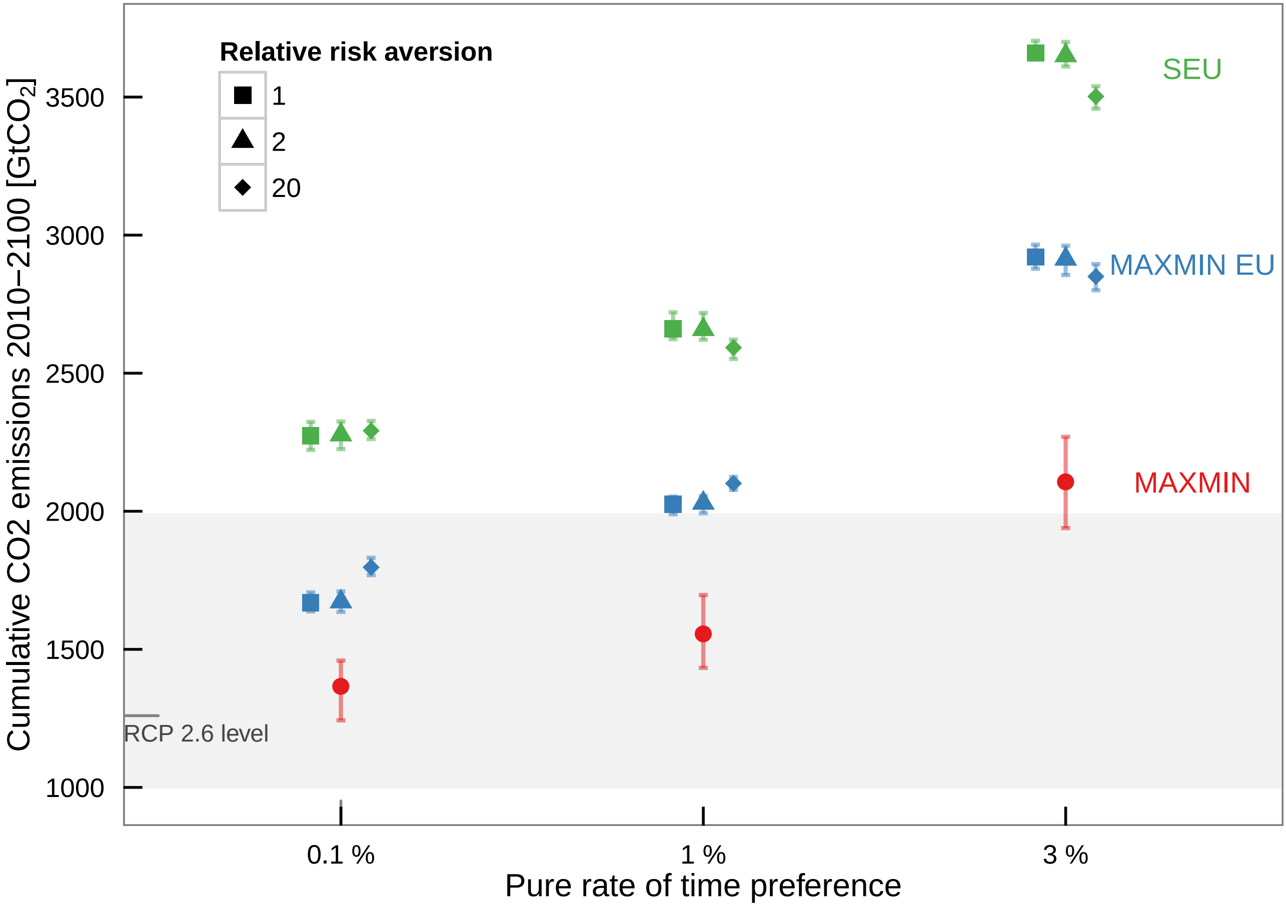
Figure 3 Selected carbon budgets according to three decision-making critera, relative risk aversion and pure rate of time preference. The shaded area represents the carbon budget range compatible with a radiative forcing of 2.6W/m^2, according to Table SPM.3 of the IPCC AR5 from WGI. The RCP2.6 carbon budget is also shown. Error bars represent the 90% confidence interval.



CO2
0
Scenario database from AR5 WGIII
750 model simulations
Mitigation cost
carbon budget
GHG scenarios
Probabilistic mitigation costs
T
CMIP5 results for AR5 WGI
38 climate models, 4 RCPs
Decision criteria
Intertemporal preferences, risk attitude, ambiguity
Carbon budget selection
Probabilistic temperature projections
Probabilistic economic impacts
Economic impacts from AR5 WGII
20 global impact estimates
3 prior damage functions
Impact cost
T



Carbon budget =1000 GtCO2
3000 GtCO2
5000 GtCO2
World GDP per capita (including mitigation and damage) [2010 level=1]
1
2
3
4
5
6
2020
2040
2060
2080
2100
Boxplot statistics
99%
90%
75%
50%
mean
25%
10%
1%
Damage function
Quadratic
Exponential
Sextic



Relative risk aversion
1
2
20
SEU
MAXMIN EU
MAXMIN
RCP 2.6 level
Cumulative CO2 emissions 2010−2100 [GtCO$_2$]
3500
3000
2500
2000
1500
1000
0.1 %
1 %
3 %
Pure rate of time preference

Managing Catastrophic Climate Risks under Model Uncertainty Aversion*

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Abstract

We propose a robust risk management approach to deal with the problem of catastrophic climate change which incorporates both risk and model uncertainty. Using an analytical model of abatement, we show how aversion to model uncertainty influences the optimal level of mitigation. We disentangle the role of preferences from the structure of model uncertainty, which we define by means of a simple measure of disagreement across models. With data from an expert elicitation about climate change catastrophes, we quantify the relative importance of these two effects and calibrate a numerical integrated assessment model of climate change. The results indicate that the structure of model uncertainty, and specifically how model disagreement varies in abatement, is the key driver of optimal abatement, and that model uncertainty warrants a higher level of climate change mitigation.

Keywords: Climate Change, Catastrophe, Model Uncertainty, Ambiguity, Non-expected Utility, Catastrophe

JEL Classification: D81, Q54

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1 Introduction

This paper applies recently developed tools from decision theory and expert assessment data to discuss abatement strategies in the case of climate change under deep uncertainty. We distinguish the notion of *risk* –which characterizes situations in which probabilities of a random event are perfectly known– from the broader notion of *(Knightian) uncertainty* (also called ambiguity) –which characterizes situations in which some events do not have an obvious, unanimously agreed probability assignment (Ghirardato et al., 2004).[1] More specifically, we focus on the notion of *model uncertainty* that corresponds to situations in which different data-generating mechanisms or models are considered as possible or plausible by the decision maker (DM). Throughout the paper, we consider the notion of *model* taken in its statistical sense, meaning that it is defined as a probability distribution over a sequence (or here over states of the world). Different models may exist for example because too little information is available, because different predictions exist (depending on different datasets, different techniques, etc.), or because the decision is based on the advice of experts who provide different assessments of probabilities for a given event (as will be the case in our application). We present the risk management problem as an intertemporal problem of optimal abatement under the possibility of a catastrophic climate event. As it is the case with a vast range of economic problems, the climate change case illustrates particularly well a situation in which the probabilistic model is neither explicitly given, nor can be perfectly approximated or inferred with the available data and current scientific methods. Choosing among different climate policies in this situation is therefore essentially an exercise in risk management that has to be performed in a situation of deep uncertainty (Kunreuther et al., 2013). Therefore, it requires a robust decision making approach that is less sensitive to initial assumptions, is valid for a wide range of futures, and keeps options open (Lempert and Collins, 2007), rather than a formal approach that maximizes the expected utility mechanically.

More than ever before, it is now widely believed that our entire planet is undergoing climate change, and that this change is largely due to human activities (IPCC, 2013). What is less clear is how this process, which is taking place over a very long time horizon, will unfold. Based on the available observations and on the current state of knowledge, scientific experts have constructed models in order to simulate and quantify the impact of human activity on the climate system and vice versa. However, a large degree of uncertainty surrounds these models. These uncertainties arise from both the underlying climate science (and the extreme complexity of the climatic system) and our inability to perfectly capture the way our socioeconomic system would respond and adapt to climate change (Heal and Millner, 2014). This is particularly the case when we consider situations with potential catastrophic consequences, such as the collapse of the Atlantic thermoha-

[1]This definition comprises the definition of *deep uncertainty* given by Lempert et al. (2006). In this paper, we use the terms Knightian uncertainty, ambiguity and deep uncertainty interchangeably.

line circulation or the melting of the Antarctic ice sheet. Such catastrophic events have never been encountered in recent history,[2] and their likelihood of occurrence is therefore extremely difficult to assess. From a decision maker's perspective, becoming aware of such occurrences leads to the expansion of the set of admissible states of the world. Therefore, the state space and the associated probabilities need to be adjusted for making decisions when considering such "unforeseen events" (Karni and Viero, 2013). In recent years, a few studies have been undertaken to estimate these probabilities, consisting generally of experts' assessments.[3] Since climate science is currently unable to determine which of these estimates is the best or what the best combination of them is, and since these uncertainties are expected to persist even when better scientific models become available,[4] a decision maker confronted with this situation would find himself in a situation of *model uncertainty* rather than *risk*.

In view of this disagreement among experts or models, how should a rational policy decision maker proceed? If he follows the traditional Bayesian/subjective expected utility approach, he will simply aggregate the models by averaging them into a single representative one, and then use the (subjective) expected utility framework (Newbold and Daigneault, 2009). The problem with this approach is that the decision maker considers the resulting aggregated model in exactly the same way as he would consider an equivalent objective model representing a specific *risk*, and model uncertainty has therefore no impact on the decision making process. This approach has however been seriously challenged in situations of deep uncertainty. The most famous example is that of Ellsberg (1961), who showed through different experiments that the choices of individuals cannot be rationalized under the traditional Bayesian expected utility paradigm, and that individuals usually manifest aversion towards situations in which probabilities are not perfectly known. In applied economic models, some recent contributions have started applying non-expected utility frameworks (i.e. alternative models of risk preferences and beliefs, most of which replace the expected utility formulation with alternative criteria) to the problem of climate change. In particular, these include applications of the smooth ambiguity model by Lange and Treich (2008), who provide comparative statics results of the role played by ambiguity in a simple two-period parametric model, Millner et al. (2013), and Lemoine and Traeger (2014), both of whom propose numerical models under ambiguity aversion. Other contributions include applications of the macroeconomic technique of robust control (Hansen

[2]These phenomena have been referred to as "tipping elements" because they imply abrupt climate change occurring "when the climate system is forced to cross some threshold, triggering a transition to a new state" (Lenton et al., 2008). Their corresponding critical point at which the future state of the system is qualitatively altered is called a "tipping point".

[3]Remark that these experts' assessments may be the result of using different climatic models, different physical parameters, different methodologies, or different databases.

[4]Even if climate scientists have recently made great deal of progress in understanding and describing the physical mechanisms involved in the climate change phenomenon, many uncertainties still remain. Some of them will eventually be resolved with future scientific progress, while others may be in the realm of "unknowable" (Pindyck, 2013a) or "unquantifiable" (Heal and Millner, 2014).

and Sargent, 2008) by Athanassoglou and Xepapadeas (2012), who consider an analytical pollution control problem, and by Rudik (2015), who applies the concept in an integrated assessment model including learning.[5] Finally, a different approach is taken by Drouet et al. (2015) who use the results of the most recent assessment of the IPCC to numerically disentangle model uncertainty and risks about mitigation costs, climate dynamics and (continuous) climate damages. For what concerns the inclusion of catastrophic damages into integrated assessment models, our paper also extends the work of, among others, Gjerde et al. (1999), who show that taking into account a risk of catastrophe provides a rationale for current emission control, and Keller et al. (2004) and Lontzek et al. (2012) who model a collapse in ocean circulation as a permanent shock to the production function, and show that the optimal policy should be associated with immediate limitations on emissions.

In this paper, we take a step further in the direction of understanding the theoretical mechanisms underlying the results obtained in this literature, by applying the most recent robust tools developed in decision theory (Cerreia-Vioglio et al., 2013b; Marinacci, 2015). We consider an alternative to expected utility models which allows to incorporate both risk and model uncertainty. More specifically, we study the impact of model uncertainty aversion on optimal abatement policy. Our contribution is severalfold. We develop a two-period model of emission abatement with an endogenous probability of catastrophic climate change, which allows us to disentangle the contribution of preferences and the structure of model uncertainty on the level of first-period abatement. We show that a simple measure of the disagreement across models or experts is a sufficient statistic for determining the structure of model uncertainty that matters for abatement. We apply our theoretical results using an actual assessment of a major catastrophic climatic event with data from a recent experts' elicitation. Finally, we extend a widely used integrated assessment model (IAM) of climate change (DICE, Nordhaus (1993)) to include a tipping element in the climate response, in a framework where well-defined probabilities are unknown. This allows us to quantify the impact of deep uncertainty on the optimal level of emission abatement, addressing one of the criticisms of IAMs which have been recently highlighted by Stern (2013), Pindyck (2013a,b, 2015), and Kunreuther et al. (2013). Our broader finding is that in most situations, a robust climate strategy implies stronger mitigation policies. In that sense, we show that deep uncertainty cannot be taken as an excuse for inaction, making a clear link to the precautionary principle.[6] We show that both preferences over model uncertainty –measured by ambiguity prudence– and the structure of the model uncertainty –measured by the decrease of the degree of model disagreement in abatement– determine

[5]Note that the preferences used by Hansen and Sargent (2008) can be seen as a special case of the smooth ambiguity preferences. In their case, the ambiguity function is of the exponential or *constant absolute ambiguity aversion* type (Cerreia-Vioglio et al., 2011; Marinacci, 2015).

[6]The precautionary principle states that: "When an activity raises threats of harm to human health or the environment, precautionary measures should be taken even if some cause-and-effect relationships are not fully established scientifically" (1998 Wingspread Statement on the Precautionary Principle).

the optimal abatement level. The data we use from expert elicitations indicate that it is the latter effect which is by far the most important, given that the disagreement across models or experts increases in global mean temperature.[7] Finally, the reformulated integrated assessment model allows to generate quantitative estimates of the impact of risk and model uncertainty aversion on optimal emission reductions. Compared to the commonly used expected utility framework, model uncertainty raises abatement significantly. Our broader policy result corroborates the findings of the recent strand of research which has emphasized the importance of deep uncertainty and tipping points in quantitative climate policy making (Lemoine and Traeger, 2014; Lontzek et al., 2012; Gjerde et al., 1999; Weitzman, 2012, 2009; van der Ploeg and de Zeeuw, 2014; Lempert and Collins, 2007; Drouet et al., 2015).

Our results can be read in both positive or normative terms. While we recognize the existence of a debate about the normative status of non-expected utility models, and the predominance of the expected utility theory paradigm for normative purposes in decision making, we here follow the claim that there is nothing irrational about violating Savage's (1954) axioms in situations of deep uncertainty (Gilboa et al., 2008, 2009, 2012; Gilboa and Marinacci, 2013).[8] The non-Bayesian framework we adopt is thus compatible with a normative assessment of optimal policies.

2 A simple model of optimal abatement under model uncertainty

To investigate the effects of different types of uncertainty on emission abatement decisions, we construct a simple economic model of optimal abatement with two periods: *today* and *the future*. During the first period, the decision maker chooses a level of abatement a that is undertaken at cost $c(a)$. This abatement reduces available disposable income in such a way that consumption in period 1 is given by $C_1 = w_1 - c(a)$, where w_1 is the level of income of the first period. In the future, there are two possible categories of states of the world. One is catastrophic: the environment is severely affected so that the consumption in the second period C_2 is given by $w_2 - L_s$, where w_2 is the deterministic exogenous income, and L_s is the damage (loss) that occurs with probability π_s, conditionally on the fact that a catastrophe occurred (i.e. $\forall s \in S$, where S represents those catastrophic, or *unfavorable* states). The other is one in which no catastrophe occurs, so that consumption is w_2 (*favorable* state). The probability $p(a)$ that such a catastrophic event will occur is

[7] Climate data allow to predict low level of future warming with more confidence than high level of warming: "once the world has warmed by 4 °C, conditions will be so different from anything we can observe today that it is inherently hard to say when the warming will stop" (Allen and Frame, 2007).

[8] In situations where information is scarce, alternative decision-theory models may on the contrary perform better in the sense that they have better explanatory power or are able to provide better predictions and guidelines.

endogenous and depends on the level of abatement chosen in the first period.[9] Consumption in the second period can therefore take $|S|+1$ different values, and the abatement effort in the first period is the only choice variable in this model. It is conceptualized as an investment to reduce the risk of a catastrophic event that is difficult to compensate by ordinary savings (rather than an instrument used to optimally smooth consumption over time). As in most environmental economic models under uncertainty, intertemporal utility is assumed to be time-separable, and future utility is discounted by a factor $\beta \in (0,1]$.

Model uncertainty is introduced by relaxing the assumption that all the elements of the maximization program are objectively known or commonly agreed upon, so that the probability model over future consumption is no longer unique. We assume that a true climatic process is in place and generates observations, but that this true process –and the probability model representing it– is unknown to the decision maker. The observations generated by this model are however available and used by *experts* (scientists, climatologists, physicists, etc.) to construct predictive models that belong to a class M. The true process is assumed to belong to this class of models, and elements of M are interpreted by the DM as possible alternative models that could be selected by nature to generate observations. These possible models have a Waldean interpretation in the sense that the class M is regarded as a datum of the decision problem (Wald, 1950). This implies that the models have to be consistent with objectively available information (note however that the information must be incomplete, otherwise M would be a singleton). This set therefore contains all the information the DM considers as "credible" in the sense that "states that are not given any weights by any of the relevant probability distributions are simply irrelevant" (Gajdos et al., 2008). We assume there are n different models (or experts). These different models P_θ are indexed by a parameter $\theta \in \{1, 2, ..., n\}$, so that $M = \{P_\theta\}_{\theta \in \{1,...,n\}}$. Each P_θ describes a possible distribution (i.e. a possible *risk*) on second-period consumption: $\tilde{C}_2(a, \theta)$ (in what follows, only the probability of catastrophe will depend on θ). We also assume that the decision maker has a prior probability measure over the set of possible models, that is, $\tilde{\theta}$ has a probability distribution $q = (q_1, q_2, ..., q_n)$, so that $\tilde{\theta}$ takes value θ with probability q_θ. This second order distribution $\tilde{\theta}$ reflects model uncertainty in the sense that the DM does not know which of the models P_θ is the true or the most accurate one, and associates a subjective weight q_θ with each of them.[10]

In what follows, we consider different criteria for decision making under climate model

[9]This type of model with endogenous probability to model mitigation is referred to as *self-protection* models in the risk literature. An alternative is to consider the case of adaptation or *self-insurance* in which the loss in the second period depends on the abatement level. Given the limited scope for adaptation in reducing catastrophic impacts, we decided not to consider this latter case in this paper. It can however be shown that our main results hold and would even be reinforced by the presence of adaptation to the catastrophe or standard continuous damages in our framework (Berger, 2014a).

[10]As mentioned earlier, a parallelism can be made between the uncertainty that follows this decomposition into model (or epistemic) uncertainty and risk (also called aleatory or physical uncertainty) and what is generally referred to in the decision theory literature as *ambiguity* (i.e. situations in which "a decision maker does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem he is facing" (Cerreia-Vioglio et al., 2013a)).

uncertainty, and compare them in terms of optimal abatement. In the Appendix B, we discuss the case of uncertainty about the economic impacts of a climate catastrophe (i.e. the size of L_s), showing that our results carry over (and are even strengthened) in this case. While different existing models of ambiguity aversion could have been adapted to the presence of objective information (Marinacci, 2015), we focus on the smooth model of model uncertainty aversion for its ability to characterize the notion of ambiguity neutrality, its mathematical convenience, and because it encompasses many of the alternative criteria as special or limit cases. Nonetheless, we explore alternative criteria, such as maxmin, in the online Supplemental Material, showing that they entail qualitatively similar results.

The traditional approach for addressing a problem in which the true distribution is unknown is to consider that agents use their probabilistic beliefs over the source of uncertainty in an expected utility maximization framework. Cerreia-Vioglio et al. (2013b) are the first to provide a decision theoretic derivation of this type of deep uncertainty presented in two layers. In particular, they enrich the standard Savage framework in the presence of objective information, and show that preferences satisfying Savage's axioms may be represented, in the context of our abatement model, by:

$$W_{SEU} = v(w_1 - c(a)) + \beta \mathrm{E}_\theta \mathrm{E} v(\tilde{C}_2(a, \tilde{\theta})). \tag{1}$$

In this expression, v is the per-period von Neumann-Morgenstern (vNM) utility function reflecting both the decision maker's attitude towards risk and his desire to smooth consumption over time,[11] E_θ is the expectation operator taken over prior distribution $\tilde{\theta}$, that is, $\mathrm{E}_\theta X(\tilde{\theta}) = \sum_{i=1}^{n} q_i X(i)$, and E is the expectation operator over second period consumption in the different states of the world, conditional on model θ. This representation is called *Classical Subjective Expected Utility* (SEU) because it incorporates key objective pieces of information in Savage's subjective framework. In the context of this paper, the second-period expected utility for a given model θ may be written as:

$$\mathrm{E} v(\tilde{C}_2(a, \theta)) \equiv p(a, \theta) \sum_{s \in S} \pi_s v(w_2 - L_s) + (1 - p(a, \theta)) v(w_2). \tag{2}$$

where we denote by $p(a, \theta)$ the probability of catastrophe as a function of abatement for model θ. For each prior distribution q, there exists an equivalent predictive distribution $\tilde{C}_2(a, \bar{\theta})$ such that $\mathrm{E}_\theta \mathrm{E} v(\tilde{C}_2(a, \tilde{\theta})) = \mathrm{E} v(\tilde{C}_2(a, \bar{\theta}))$, and it is therefore clear that the reduced form of representation (1) is nothing but the original Savagian subjective expected utility. The decision problem under uncertainty is then reduced to a simple decision problem under risk where the beliefs are subjective. On the other hand, when M is a singleton (i.e. when there is only one model everyone agrees on), there is no longer model uncertainty, so

[11] The two features could easily be disentangled using Kreps and Porteus (1978)/Selden (1978) preferences. For the sake of expositional clarity and simplicity, we only consider this specification in the quantification part of the paper (see Section 4).

that the risky second period consumption is $\tilde{C}_2(a)$ and we are back to the classical vNM expected utility model. These different representations of the problem are observationally equivalent to someone who is not aware of the presence of objective information deriving from different experts' models.

In what follows, we consider the subjective expected utility representation as a benchmark. The economic problem of finding the level of abatement a^*_{SEU} that maximizes program (1) is easy to solve.[12] This level is implicitly given by equalizing the marginal cost and benefit of abatement:

$$v'(w_1 - c(a^*_{SEU}))c'(a^*_{SEU}) = -\beta \frac{\partial \mathrm{E}_\theta \mathrm{E}v(\tilde{C}_2(a^*_{SEU}, \tilde{\theta}))}{\partial a}. \tag{3}$$

While the classical subjective expected utility framework has the advantage of being easily tractable, it is unable to take into account different attitudes towards different types of uncertainty that surround the economics of climate change. We now introduce different attitudes towards different types of uncertainty. In order to investigate the relationship between risk aversion and model uncertainty aversion, we consider a criterion in which the functional representing the agent's preferences towards model uncertainty is smooth and hence everywhere differentiable. Breaking the equal treatment of different uncertainties and letting v represents attitude towards risk, and h attitude towards model uncertainty, we can write the smooth criterion (Marinacci, 2015) to be maximized as:

$$W_{\mathrm{Smooth}} = v(w_1 - c(a)) + \beta(v \circ h^{-1}) \left(\mathrm{E}_\theta (h \circ v^{-1}) \left(\mathrm{E}v \left(\tilde{C}_2(a, \tilde{\theta}) \right) \right) \right). \tag{4}$$

This expression can be written equivalently as

$$W_{\mathrm{Smooth}} = v(w_1 - c(a)) + \beta v \left(CE \left(ce(a, \tilde{\theta}) \right) \right), \tag{5}$$

where ce and CE represent both a certainty equivalent:

$$ce(a, \theta) \equiv v^{-1} \left(\mathrm{E}v(\tilde{C}_2(a, \theta) \right) \quad \text{and} \quad CE(ce(a, \tilde{\theta})) \equiv h^{-1} \left(\mathrm{E}_\theta h(ce(a, \tilde{\theta}) \right). \tag{6}$$

The first, $ce(a, \theta)$, corresponds to the certainty equivalent of wealth in the second period, if the abatement level is a and the expert's model considered is P_θ. Under model uncertainty, θ itself takes on different values, and so does the certainty equivalent $ce(a, \theta)$, which is computed conditionally on θ. A second-order certainty equivalent of these first-order certainty equivalents is then defined as CE by combining all models $\theta \in \{1, 2, ..., n\}$. The SEU reprensetation (1) is then obtained in the special case in which the two certainty equivalents are evaluated using the same function v, so that the attitudes towards risk and

[12]The maximization programs we consider in this paper are assumed to be convex. Sufficient conditions for concavity of (1) are that the cost function is increasing ($c'(a) > 0$) and convex ($c''(a) > 0$) in the level of abatement, and that the probability function is decreasing and convex ($p'(a) < 0$, $p''(a) > 0$). More generally, sufficient conditions for concavity of (4) may be found in Proposition 3 in Berger (2014a).

model uncertainty are exactly the same.

The smooth model uncertainty criterion is mathematically equivalent to the two-period version of the ambiguity model developed by Klibanoff et al. (2009). The significant difference is that their model, as the vast decision theoretic literature dealing with ambiguity (see Gilboa and Marinacci (2013) for an excellent survey), has been developed in a purely subjective setup, and therefore does not explicitly incorporate objective information à la Wald (1950). In particular, their representation is recovered by letting $\phi \equiv h \circ v^{-1}$ represent the ambiguity attitude. Klibanoff et al. (2005) associate ϕ being a concave function to ambiguity aversion, and call the ratio $-\frac{\phi''(x)}{\phi(x)}$ the coefficient of absolute ambiguity aversion at x, a given level of expected utility. From representation (4), ambiguity aversion would correspond to h being more concave than v, or equivalently model uncertainty aversion being stronger than risk aversion. Unsurprisingly, in the special case in which the DM manifests the same attitude towards risk and model uncertainty, the problem is reduced to the one considered by a classical subjective expected utility maximizer defined by representation (1). In this case, the decision problem may be reduced to a problem under risk.[13] The great flexibility of this decision rule, which is based on the smoothness of function h, implies different conditions in the comparative statics analysis of optimal abatement. In particular, one condition needed to sign the direction of the change resulting from higher aversion towards model uncertainty than towards risk is the notion of *ambiguity prudence* (Gierlinger and Gollier, 2008; Berger, 2014b). This concept, which is closely related to the notion of *risk prudence* introduced by Kimball (1990), corresponds to a condition under which the individual is willing to save more because of the presence of ambiguity.[14] Equivalently, it expresses the sensitivity of the optimal choice to the combination of model uncertainty and risk. The notion of ambiguity prudence in this context corresponds to *decreasing absolute ambiguity aversion*, which is the analogue of the widely accepted notion of decreasing absolute risk aversion (DARA). Formally, the notions of constant, decreasing and increasing absolute ambiguity aversion are defined depending on the monotonicity properties of the ratio $-\frac{\phi''(x)}{\phi(x)}$. In what follows, we respectively use the abbreviations CAAA, DAAA, and IAAA to denote these cases.[15] Equipped with the ambiguity prudence property, which we investigate further in the next section, we now compare the optimal level of abatement chosen by a decision maker under the smooth criterion with the one chosen under classical subjective expected utility. Firstly, let us

[13] Note also that the maxmin criterion presented in the online Supplemental Material is recovered from this formulation in the special case of infinite model uncertainty aversion.

[14] Formally, an agent is said to be ambiguity prudent if the introduction of ambiguity through a mean-preserving spread in the space of conditional second period expected utility raises his optimal level of saving (Berger, 2014b).

[15] Note that DAAA encompasses the most widely used functional forms of power and exponential ϕ (the former is usually referred to 'constant relative ambiguity aversion' (CRAA) and the latter corresponds to CAAA). It is stronger than requiring $\phi''' > 0$, but this should not be surprising, given that future utility in program (4) is represented by the ϕ-certainty equivalent of the expected utilities rather than by its expected ϕ-valuation.

recall the definition of comonotonic variables before summarizing the result in Lemma 1, which is reminiscent of Alary et al. (2013) and Berger (2014a):[16]

Definition 1. Consider two random variables X and Y that are strictly monotonic transformations of a single random variable θ, that is, $(X,Y) = (f(\theta), g(\theta))$. The random variables X and Y are anticomonotonic if f is increasing and g is decreasing in θ, and comonotonic if f and g are both increasing or decreasing in θ.

Lemma 1. *Assume that model uncertainty aversion is higher than risk aversion. In the optimal abatement model characterized by the maximization of program* (4)*, DAAA (resp. IAAA) is sufficient to raise (resp. decrease) the optimal abatement if* $\mathrm{E}v(\tilde{C}_2(a^*_{SEU},\theta))$ *and* $\partial \mathrm{E}v(\tilde{C}_2(a^*_{SEU},\theta))/\partial a$ *are anticomonotonic (resp. comonotonic).*

Proof. See Appendix A.1 □

Lemma 1 tells us that the total effect on abatement not only depends on the ambiguity prudence condition, but also on a second factor that concerns the way the second period expected utility and the marginal benefit of abatement interact when different experts or models are considered. The question of whether the comonotonicity condition of Lemma 1 holds in practice is not trivial. However, the intuition is relatively simple. Consider the case of two models with different probability curves $p(a,\theta)$ that do not cross. When the more pessimistic one (e.g. the one with lower $\mathrm{E}v(\tilde{C}_2(a^*_{SEU},\theta))$) believes that the probability of catastrophe decreases faster in abatement (e.g. a higher $\partial \mathrm{E}v(\tilde{C}_2(a^*_{SEU},\theta))/\partial a$), then the anticomonicity condition holds, and the condition of ambiguity prudence –stating that the DM is more willing to invest for the future when this future becomes more uncertain– is sufficient. In this case, it is equivalent to saying that the disagreement across models falls in abatement. For example, this would be the case if experts agreed that a high level of climate protection would give us good chances of avoiding a climate catastrophe, but disagreed on the probabilities in the case of limited mitigation and thus higher global warming. In order to gain more intuition, we now disentangle the role of preferences from the structure of model uncertainty, and study the two effects separately.

2.1 The ambiguity prudence effect

In terms of attitudes towards risk and model uncertainty, the ambiguity prudence condition turns out to be non-trivial, as summarized in the following proposition.

[16] In a recent contribution Millner et al. (2013) also proposed a model of abatement with an endogenous probability in a one-period framework similar to Alary et al. (2013). When two periods are considered, however, the comonotonicity condition only concerns the second period, and general conclusions may be drawn for the more realistic cases in which the effort exerted in the first period also reduces the ambiguity.

Proposition 1. *A decision maker exhibits DAAA if and only if his preferences towards risk, captured by function u, and model uncertainty, captured by function h, are such that:*

$$\frac{h'''}{h'} - \frac{v'''}{v'} \geq \left(-\frac{h''}{h'} + \frac{v''}{v'}\right)\left(-\frac{h''}{h'} - 2\frac{v''}{v'}\right). \tag{7}$$

Similarly, a decision maker exhibits CAAA if inequality (7) *holds with an equality, and IAAA if inequality* (7) *is reversed.*

Proof. See Appendix A.2 □

Intuitively, Proposition 1 tells us that a necessary and sufficient condition for DAAA is that the difference in downside model uncertainty and risk aversion (the left-hand side of (7)) is sufficiently high. To gain further insight about this condition, consider the following examples.

Example 1. When the isoelastic CRRA-CRMUA[17] specifications with relative risk aversion parameter ρ and relative model uncertainty aversion parameter $\mu \geq \rho$ (so that the DM is ambiguity averse) are considered, the ambiguity aversion function is given by $\phi(U) = \frac{1}{1-\mu}[(1-\rho)U]^{\frac{1-\mu}{1-\rho}}$, the coefficient of absolute ambiguity aversion is $\frac{\mu-\rho}{(1-\rho)U}$, and the DM exhibits DAAA when $\rho < 1$, CAAA when $\rho = 1$, and IAAA when $\rho > 1$.

Example 2. When the CARA-CAMUA[18] specifications are used with absolute coefficients of risk aversion and model uncertainty aversion respectively ρ and μ, the ambiguity function is $\phi(U) = -(-U)^{\frac{\mu}{\rho}}$, so that the DM always exhibits IAAA.

2.2 The convergence of agreement effect

In order to study the structure of model uncertainty, let us simplify the notation in expression (2) above, and let $w_2 - L$ with $L > 0$, be the certainty equivalent consumption in the second period when the economy is hit by a catastrophe.[19] Remember that in this case, each model P_θ describes a possible *risk* on second period consumption, which is fully characterized by $\tilde{C}_2(a,\theta) \sim [w_2 - L, p(a,\theta); w_2, 1 - p(a,\theta)]$. An illustration of possible different models is depicted in the first row of Figure 1.

To further characterize the change in the optimal abatement decision, we now define the notion of *degree of model disagreement.* It is a measure of the variability across models (or, equivalently, of the disagreement among experts).

[17] A utility function has the CRRA (constant relative risk aversion) property if it takes the form $v(x) = \frac{x^{1-\rho}}{1-\rho}$, where ρ is the coefficient of relative risk aversion (note that when $\rho = 1$, it collapses to $v(x) = \ln x$). CRMUA (constant relative model uncertainty aversion) is defined similarly for function h, in the sense that $h(x) = \frac{x^{1-\mu}}{1-\mu}$, where μ represents the coefficient of relative model uncertainty aversion.

[18] A utility function exhibits CARA (constant absolute risk aversion) if it has the form $v(x) = -e^{-\rho x}$, where ρ is the coefficient of absolute risk aversion. CAMUA (constant absolute model uncertainty aversion) is defined analogously, so that $h(x) = -e^{-\mu x}$, where μ is the coefficient of absolute model uncertainty aversion.

[19] More precisely, this certainty equivalent is implicitly defined by: $\sum_{s \in S} \pi_s v(w_2 - L_s) = v(w_2 - L)$.

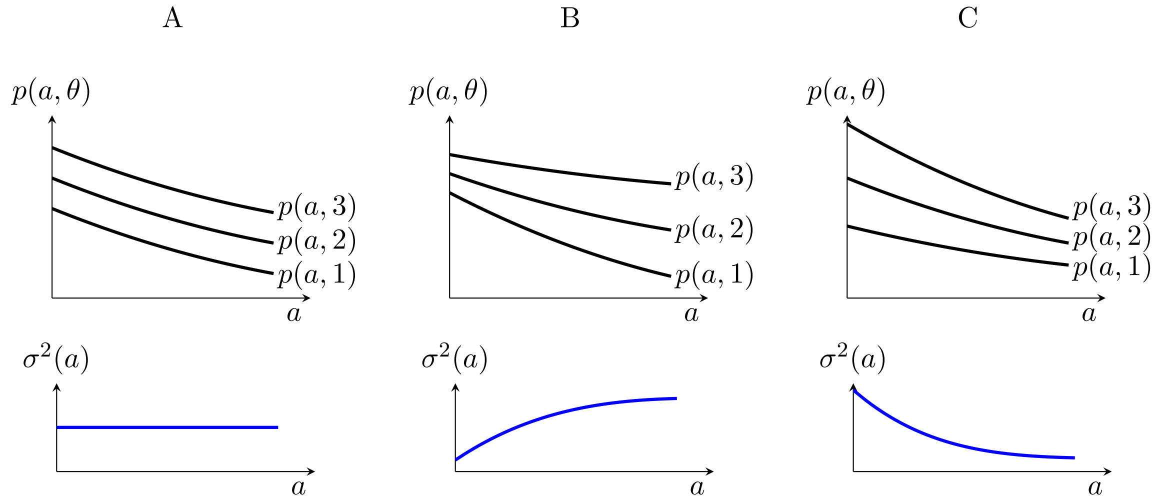

Figure 1: Different models or experts $p(a, \theta)$ as functions of the abatement level a, under constant *(column A)*, increasing *(column B)*, or decreasing *(column C)* degree of model uncertainty in abatement.

Definition 2. For any set of probability functions $\{p(a, \theta)\}_{\theta \in \{1,...,n\}}$ characterizing models $\{P_\theta\}_{\theta \in \{1,...,n\}}$, the *degree of model disagreement* is given by:

$$\sigma^2(a) := \text{Var}_\theta[p(a, \theta)], \quad \text{for any given level of abatement } a.$$

The degree of model disagreement is illustrated in the second row of Figure 1. As can be seen, it can be (but is not limited to) constant (column A), increasing (column B), or decreasing (column C) in the level of abatement. In what follows, we will focus on the case where the degree of model disagreement is a monotonic function of abatement.[20] We will refer to *convergence of agreement* a situation in which the degree of model disagreement is decreasing. Using this simple metric, we can now relate the results of Lemma 1 to the structure of model uncertainty. First, note that conditional on the true model being P_θ, we can write:

$$\text{E}v(\tilde{C}_2(a^*_{SEU}, \theta)) = v(w_2) - p(a, \theta)\Big[v(w_2) - v(w_2 - L)\Big] \tag{8}$$

$$\frac{\partial \text{E}v(\tilde{C}_2(a^*_{SEU}, \theta))}{\partial a} = -p_a(a, \theta)\Big[v(w_2) - v(w_2 - L)\Big], \tag{9}$$

where $p_a \equiv \frac{\partial p}{\partial a}$. Lemma 1 therefore tells us that a DM exhibiting DAAA will always choose to abate more if $p(a, \theta)$ and $p_a(a, \theta)$ are anticomonotonic in θ, as it is for example the case in column C of Figure 1. In this case, the degree of model uncertainty will intuitively be decreasing in abatement since abatement decreases the probability of catastrophe more

[20] Remark that this assumption is less restrictive than the one requiring the set of models $\{P_\theta\}$ to be monotonic in θ for all levels of abatement equivalent to the one used in Alary et al. (2013) and Berger (2014a). In particular, our assumption does not require probability curves not to cross each other.

strongly in more pessimistic models. The sufficient condition of Lemma 1 is however very restrictive, and Proposition 2 below tells us that the comonotonicity property does not necessarily have to hold for all the models considered. Instead, a simple and weaker condition on the degree of model disagreement can be used to determine the direction of the change induced by ambiguity aversion:

Proposition 2. *The degree of model disagreement* $\sigma^2(a)$ *is decreasing (resp. increasing) in abatement if and only if* $\mathrm{Cov}_\theta\Big(p(a,\theta); p_a(a,\theta)\Big) \leq$ *(resp.* $\geq$*)* 0.

Proof. See Appendix A.3 □

With this intuition in mind, we can now introduce our main result, which does not require the relatively strong condition of comonotonicity.

Proposition 3. *In the optimal abatement problem under model uncertainty,*

(i) a decision maker exhibiting CAAA always chooses to abate more (resp. less) than an SEU maximizer if the degree of model disagreement decreases (resp. increases) with abatement.

(ii) a decision maker exhibiting strict DAAA (resp. IAAA) always chooses to abate strictly more (resp. less) than an SEU maximizer if the degree of model disagreement decreases (resp. increases) or is constant in abatement.

This proposition tells us that if higher abatement leads to a reduction in the *degree of model disagreement*, a positive incentive is generated to abate more in the first period. Intuitively, the degree of model disagreement will be decreasing in abatement if abatement on average decreases the probability of a catastrophe more strongly in pessimistic models. This structural effect has however to be added to the ambiguity prudence effect to determine the direction of the total change in the abatement level. Ultimately, whether experts' disagreement decreases in abatement, and the extent to which the model structure effect interplays with ambiguity prudence in determining the optimal level of mitigation can be answered only numerically. In the next section we bring the model to the data and analyze the direction and magnitude of both effects.

3 Empirical evidence and expert judgments

The question of whether the degree of model disagreement is increasing or decreasing in the level of abatement is essentially an empirical one. In this section, we use the results of a recently published study to assess whether the conditions obtained from our theoretical model are met in practice. In particular, we study separately the two effects of ambiguity prudence and convergence of agreement we described in the previous section.

We use the data of Zickfeld et al. (2007). Their study presents the results from interviews with 12 leading climate scientists about the risk of a collapse of the Atlantic

Meridional Overturning Circulation (AMOC, also called Thermohaline Circulation) due to global climate change.[21] Specifically, the authors elicited the experts' probabilities[22] that a collapse of the AMOC will occur or will be irreversibly triggered as a function of global mean temperature (GMT) increase realized by the year 2100. These probabilities are reproduced and approximated in a least-squares sense using a power function of the type $P(T) = k_1 T^{k_2}$, where T represents the change in global mean temperature, and k_1 and k_2 are the best fit coefficients, in the above part of Figure 2. Note that for $T = 0$, the probability of catastrophe $P(T)$ is set to zero for all experts. As it can be seen, eight

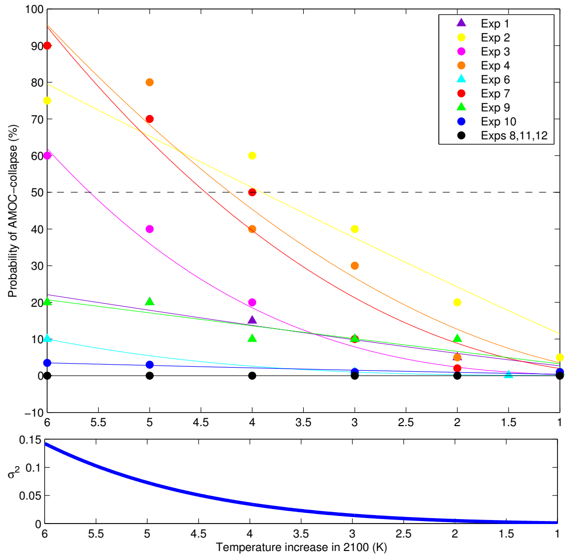

Figure 2: *Above:* Experts' probabilities as a function of global mean temperature in 2100. *Below:* Degree of scientific model disagreement (σ^2)

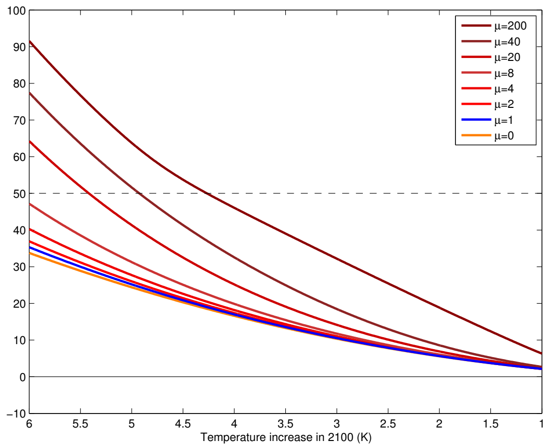

Figure 3: Distorted probabilities of the catastrophe. Specifications: v: CRRA (ρ=1), and h: CRMUA (μ)

experts[23] assessed a non-zero probability of this catastrophic event. For an increase of 2 degrees Celsius in 2100 relative to 2000, four experts assessed a probability of a collapse $\geq 5\%$, while for a warming of 4°C, three experts assigned a probability $\geq 40\%$. Finally, if

[21]The AMOC is a major current in the Atlantic Ocean that transports heat energy from the tropics and Southern Hemisphere towards the North Atlantic. Changes in this ocean circulation could have an important impact on many aspects of the global climate system, including changes in the carbon-cycle. A *collapse* of the AMOC is defined in Zickfeld et al. (2007) as a "reduction in AMOC strength by more than 90% relative to present-day". Such an event may potentially have catastrophic consequences such as changes in sea level in the Atlantic area north up to 1 meter (Zickfeld et al. (2007) Fig.7), reductions in crop production or water availability with consequent impacts (Table 12.4, IPCC (2007)). The list of scientists selected for this study can be found in Zickfeld et al. (2007). It includes experts with different scientific backgrounds (observationalists, palaeoclimatologists, modelers), geographic origins and schools of thought. These experts were selected based on different criteria (authors' knowledge of the field, review of recent publications, advice from scientists in the field).

[22]Expert elicitations is a tool for systematically gathering and projecting scientific information in complex policy problems that is "increasingly recognized to play a valuable role for informing climate policy decisions" (Kriegler et al., 2009).

[23]Note that expert 5 did not answer the question.

the increase in global warming reaches 6°C, the probability is 90% for two experts, $\geq 50\%$ for four, and $\geq 10\%$ for six experts. These curves represent the different probability functions $p(a, \theta)$ we introduced in Section 2, given that the abatement of GHG emissions lowers expected global mean temperature. Although the link between cumulative emissions and temperature increase has been shown to be robustly described by a linear relationship (Matthews et al., 2009; IPCC, 2013), the magnitude of the so-called carbon-climate response describing this relationship remains uncertain. In our framework, we have so far neglected this additional source of uncertainty. In the Supplemental Material provided online, we allow for different values of carbon-climate response and show that our results are robust to this alternative source of uncertainty.

In Figure 3, we provide the distorted probability functions for different values of model uncertainty aversion. Formally, this notion is defined as follows.

Definition 3. The *distorted probability* $\hat{p}(a)$ is the probability that would be equivalently considered under expected utility, and that is defined as:

$$\hat{p}(a)v(w_2-L)+(1-\hat{p}(a))v(w_2) = (v \circ h^{-1})\Big\{\mathrm{E}_\theta (h \circ v^{-1})\Big\{p(a,\tilde{\theta})v(w_2-L)+(1-p(a,\tilde{\theta}))v(w_2)\Big\}\Big\}. \tag{10}$$

Note that for an individual exhibiting an equal attitude towards risk and model uncertainty, the distorted probability corresponds to the predictive probability of catastrophe: $\bar{p}(a) \equiv \mathrm{E}_\theta p(a, \tilde{\theta})$. On the contrary, under the smooth model uncertainty aversion criterion, the DM aggregates the different models depending on his degree of model uncertainty aversion relative to his risk aversion, and acts *as if* he were an expected utility maximizer considering only the distorted probability $\hat{p}(a)$. In particular, if aversion to model uncertainty is stronger than that to risk, it must be that $\hat{p}(a) \geq \bar{p}(a)\ \forall a$, leading any ambiguity averse DM to overweight more pessimistic models. Since estimates of the potential loss L are hard to obtain, we follow van der Ploeg and de Zeeuw (2014) in assuming a 20% loss of GDP. This order of magnitude is rather speculative and is used essentially for illustrative purposes in the context of climate change, but it is based on the findings of Barro (2013), who shows that historically, catastrophes –defined as losses of at least 10% of GDP– averaged about 20% of GDP. Finally, regarding the weights of different experts, we consider a uniform prior distribution, given that we do not have any information about the "qualification" of the different experts.[24] We can now study separately the effect resulting from the degree of model disagreement (convergence of agreement effect) and the one resulting from the attitude towards model uncertainty (ambiguity prudence effect).

[24]This view is supported by Zickfeld et al. (2007), who write "the process of choosing experts for inclusion in this study is fundamentally different from the process of sampling to estimate some uncertain value such as a physical quantity, or polling the public to predict the results of an election. The route to scientific truth is not a matter of voting. One of the outliers among the respondents may be correct, and those who appear to be in close agreement may all be wrong".

Let us begin with the former. The lower panel of Figure 2 tells us that the degree of model disagreement for the AMOC collapse is decreasing in the level of abatement. To compute the distorted probabilities in Figure 3, we use a utility function v of the type CRRA with a parameter of relative risk aversion $\rho = 1$ (i.e., log utility), and a function h of the CRMUA form, with a model uncertainty aversion parameter μ. From the properties of the CRRA-CRMUA functions discussed above, the DM exhibits CAAA, so that there is no ambiguity prudence effect. The total effect on abatement can therefore be entirely attributed to the decrease in the degree of model disagreement. For $\mu = 1$, the individual is ambiguity neutral and maximizes his expected utility by considering only the probability depicted in blue. When $\mu = 0$, the DM is ambiguity loving and acts as if he considered more optimistic experts, while when μ increases, more weight is attached to more pessimistic experts, and the probability of catastrophe increases for any fixed level of abatement. What Figure 3 indicates is that not only is the distorted probability of catastrophe higher when $\mu > 1$, but so is the slope of the distorted probability functions, therefore making abatement marginally more desirable. This change in the marginal benefit of abatement induces the DM to opt for a higher abatement level.

In order to isolate the ambiguity prudence effect, we artificially construct three different probability laws representing experts' assessments of the AMOC collapse (above part of Figure 4). The probability laws are constructed in such a way that, by considering a uniform prior over experts, an EU maximizer chooses exactly the same amount of abatement as in the case where he considers the data from Zickfeld et al.'s (2007) probabilities presented in Figure 2. Since these probability laws are perfectly parallel, the degree of model disagreement $\sigma^2(a)$ is constant in abatement (below part of Figure 4). The effect of higher aversion towards model uncertainty than towards risk on optimal abatement in this case therefore depends exclusively on the DM's ambiguity prudence attitude. In particular, from our model we know that the DM abates less if he exhibits IAAA, exactly the same amount if he exhibits CAAA, or chooses to abate more if he manifests DAAA. To quantify the importance of this effect, we present the distorted probabilities with different specifications of the functions v and h in Figure 5. We consider the CRRA-CRMUA specification described above, spanning rather extreme values of relative risk aversion η and model uncertainty aversion μ. As before, when $\rho = \mu$ the DM acts as an expected utility maximizer and considers the average probability of catastrophe depicted in blue. When model uncertainty aversion is higher than risk aversion ($\mu > \rho$), he either considers the red (DAAA), the blue dotted (CAAA) or the brown (IAAA) distorted probability (see insets). The lower part of Figure 5 shows that the difference between the distorted probabilities and the simple average, $\Delta(a) := \hat{p}(a) - \bar{p}(a)$, is respectively constant, decreasing or increasing in abatement when CAAA, DAAA or IAAA is considered. This gives an ambiguity averse individual manifesting DAAA an incentive to abate more in order to prevent the realization of the bad state in the future, since the absolute slope of the

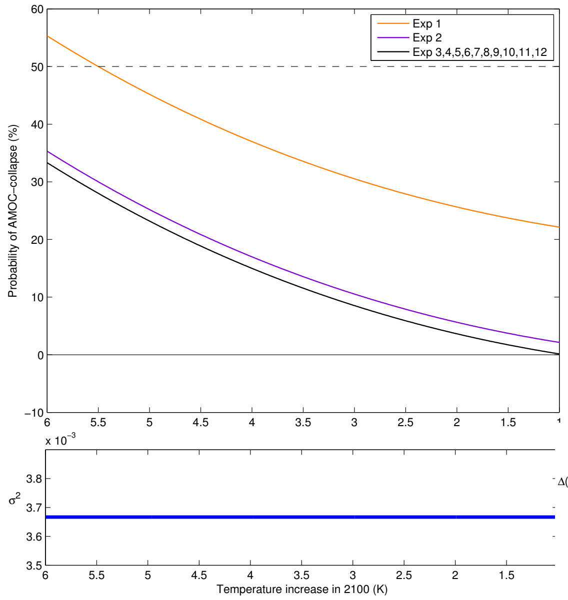

Figure 4: *Above:* Artificial experts' probabilities as a function of global mean temperature in 2100. *Below:* Degree of model disagreement ($\sigma^2(a)$)

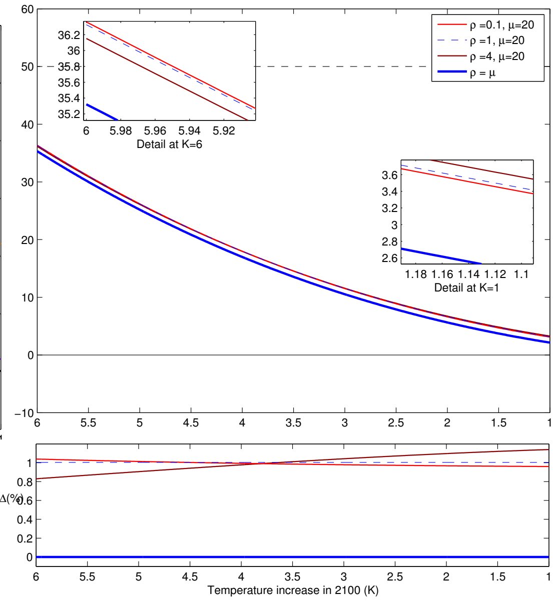

Figure 5: *Above:* Distorted probabilities of catastrophe for different specifications of v: CRRA (ρ) and h: CRMUA (μ). *Below:* Difference ($\Delta(a)$) between the distorted probability and the probability law considered under SEU

probability curve (and hence the marginal benefit of abatement) is always higher in this situation than under expected utility or under CAAA. However, although the direction of the effect is the same as predicted by our model, its magnitude appears to be small, with discernible difference only for very high values of model uncertainty aversion.[25] This provides an empirically grounded assessment of the relative importance of the structure versus the attitude toward model uncertainty, showing that the former effect –namely the convergence of model agreement– has a bigger impact on the optimal climate policy decision. In order to provide a quantitative assessment of the combined effects of model uncertainty on optimal abatement, we now apply our framework to a general equilibrium model of climate change economics.

4 Quantification using an integrated assessment model

In order to quantify the theoretical predictions, we implement the model developed in this paper using the data of Section 3 in the most widely used integrated assessment model for the analysis of climate change, the DICE model (Nordhaus, 1993). DICE (Dynamic Integrated Climate and Economy) is a numerical optimal growth model à la Ramsey,

[25]It may be shown that this result of almost no ambiguity prudence effect is robust to a reasonable range of GDP losses.

which integrates emissions and their mitigation in the production function, and which provides climate change feedback on the economy through climate and impact modules.[26] We extend the DICE model by reformulating it as a stochastic control problem, and by implementing the endogenous possibility of a climate catastrophe based on the estimated experts' probability functions. Section 2 in the online Supplemental Material provides a more detailed description of the model. Following Zickfeld et al.'s (2007) expert elicitation, we consider the case where the uncertainty is resolved at one single point in time, in the year 2100. That is, after the year 2100, either the catastrophe has hit the economy, leading to the crossing of a tipping point, or not. In the catastrophic state, an irreversible damage occurs in that an additional 20% of baseline GDP is lost for the remaining time horizon. This means that we extend the DICE damage function that expresses the economic impacts of climate change D in percent of GDP as the following random variable:

$$\tilde{D}_\theta(T) \sim \left[\kappa_1 T + \kappa_2 T^{\kappa_3} + L, p_\theta(T); \kappa_1 T + \kappa_2 T^{\kappa_3}, (1 - p_\theta(T))\right], \tag{11}$$

where T is the change in global mean temperature relative to the preindustrial level, and p_θ is the probability of suffering an additional catastrophic loss L, as given by expert θ.[27] The term $\kappa_1 T + \kappa_2 T^{\kappa_3}$ on the right-hand side of expression (11) represents the standard DICE damage function. The default calibration of $\kappa_1 = 0, \kappa_2 = 0.00267$, and $\kappa_3 = 2$ for example yields a standard damage estimate of 5.4% of GDP for a 4.5°C temperature increase. The loss due to a catastrophic event ($L = 20\%$) adds onto the standard damage function and occurs with a probability that depends on the temperature increase attained in the year 2100. Finally, while so far we assumed that the elasticity of intertemporal substitution was equal to the inverse of the degree of relative risk aversion (an assumption that is maintained throughout the literature, see Klibanoff et al. (2009)), we disentangle these two very different normative characteristics of the decision maker to obtain a more realistic representation of preferences. To do so, we modify DICE's utility function and adapt the generalized model of Hayashi and Miao (2011) to disentangle the three concepts of risk aversion, intertemporal elasticity of substitution and model uncertainty aversion. As before, we represent risk aversion by the function v and model uncertainty aversion by h. The agent's intertemporal welfare at time t is represented by the following recursive form:

$$W_t = u^{-1}\left[(1-\beta)u(c_t) + \beta u\left(R_t\left(\tilde{W}_{t+1}(\tilde{\theta})\right)\right)\right], \tag{12}$$

[26] See Nordhaus and Sztorc (2013); Nordhaus (2014) for a description of all assumptions, equations and data used for the latest version DICE2013R of the model.

[27] An adjustment factor of 0.7°C degrees, representing the global mean increase in temperature between years 1900 and 2000 (Hansen et al., 2006), is used since the DICE standard damage function considers 1900 as a reference for temperatures.

where u characterizes the attitude towards consumption smoothing over time,[28] β is the discount factor, C_t is the consumption at time t, and $R_t(\tilde{W}_{t+1}(\tilde{\theta}))$ represents the *double certainty equivalent* defined as follows:

$$R_t(\tilde{W}_{t+1}(\tilde{\theta})) := h^{-1}\Big(\mathrm{E}_{t,\theta}(h \circ v^{-1})\Big(\mathrm{E}_t v(\tilde{W}_{t+1}(\tilde{\theta}))\Big)\Big). \tag{13}$$

In this expression, $\mathrm{E}_{t,\theta}$ is the expectation operator taken at time t over models, and E_t is the expectation operator taken at time t over future consumption, conditional on θ. For the implementation, we use a threefold isoelastic specification of the different functions: η is the inverse of the elasticity of intertemporal substitution, ρ is the constant relative risk aversion (CRRA) parameter, and μ is the degree of constant relative model uncertainty aversion (CRMUA). The main purpose of the threefold disentanglement is to allow varying risk preferences while keeping the certainty equivalent discount rate, as defined by the Ramsey rule, constant.

The enhanced DICE model allows us to quantify the impact of model uncertainty aversion on abatement in the light of the insights of the theoretical model proposed in Section 2. In particular, it shows how the combined effects of ambiguity prudence and the convergence of agreement effect impact the optimal abatement decisions. In order to do so, we compute the level of additional abatement, i.e. the extra reduction of cumulative emissions, under the possibility of a climate catastrophe, relative to that in the standard version of DICE without catastrophic climate change.[29] Figure 6 illustrates the results in terms of additional abatement realized during the period 2010-2100 for different parametrizations. For $\rho = \mu = 0$, that is for a risk and model uncertainty neutral policy maker, the only difference with respect to the standard DICE is the presence of catastrophe, which is evaluated as its expected future consumption loss. In this case, optimal abatement increases by 13.5% in the sense that the cumulative emissions are further reduced from 3813 to 3301 GtCO2, as reported in Table 1 below.

The additional abatement rises to 15%, 23%, and 42% when both risk and model uncertainty aversions are increased simultaneously ($\rho = \mu$) to 1, 4 and 10 respectively (black crosses on Figure 6). This situation of ambiguity neutrality corresponds to the Epstein-Zin-Weil version of the model. Allowing to differentiate the coefficients of risk aversion and model uncertainty aversion enables us to see that the additional abatement

[28] Similarly as what is proposed by Epstein and Zin (1989) and Weil (1990), we consider the particular case in which $u$is isoelastic, with a parameter η representing the inverse of the elasticity of intertemporal substitution.

[29] Unless stated explicitly otherwise, we keep the standard specifications of the latest version of the DICE model (see Nordhaus and Sztorc (2013); Nordhaus (2014)) unaltered. This for example means that the inverse of the elasticity of intertemporal substitution is fixed to $\eta = 1.45$ and that the pure rate of time preference equals 1.5% per year. In the standard scenario without the possibility of a catastrophe, the global temperature increase by 2100 is about C, and the global cumulative CO_2 emissions –also called the cumulative carbon budget– amount to 3813 gigatonnes of carbon dioxide (GtCO2) for the period 2010-2100 (see the last row of Table 1 hereafter). We will refer to additional abatement the further relative reductions of this carbon budget.

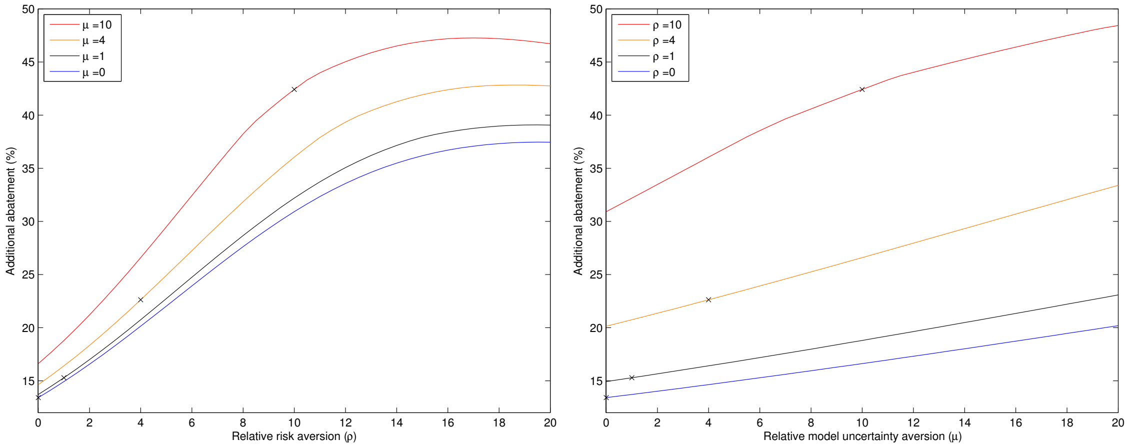

Figure 6: Additional abatement based on the modified version of DICE with the possibility of AMOC collapse for different values of relative risk aversion (ρ) and relative model uncertainty aversion (μ).

level is increasing in the risk aversion parameter ρ, though at a decreasing rate (left panel of Figure 6). For what concerns the model uncertainty aversion parameter μ, the additional abatement level monotonically increases. In terms of magnitude, the results suggest that the effect of model uncertainty aversion is about one fourth to about half of the effect of risk aversion (as can also be inferred from Figure 6): starting from the case of $\rho = \eta = 1$ with additional abatement of 15% of emissions, increasing ρ to 10 roughly doubles the effort to 32%, whereas increasing μ to 10 increases abatement to 19%. Moreover, the results of the enhanced DICE model confirm what we observed in the previous section concerning the relative importance of preferences and structure of model disagreement: since the experts' disagreement decreases in abatement, abatement always increases in the degree of model uncertainty aversion, even whenever $\rho \geq 1$.

Table 1 provides additional details of the scenario runs. In the third column, we report the social cost of carbon in 2015.[30] It is estimated to be \$17.7 per ton of CO_2 in the standard version of DICE without catastrophe and increases to \$20.4 when the possibility of a catastrophe is taken into account in a risk and model uncertainty neutral environment. It further increases to \$27 when the relative risk and model uncertainty aversion parameters $\mu = \rho = 10$ are considered. Additional results concerning the temperature increase reached in 2100 are presented in the fourth column of Table 1. As can be seen, the introduction of a potential catastrophic climate change event reduces the admitted global temperature increase in 2100 (compared to the preindustrial level) from 3.1°C in the standard version of DICE, to 2.5°C (when $\mu = \rho = 10$). Both the risk and model uncertainty aversion parameters lead to a reduction of the optimal temperature increase.[31] Finally, the two

[30] Put simply, this important concept measures the market price of emissions of GHGs. More formally, the social cost of carbon at time t is defined as the ratio of the marginal impact of emissions on welfare over the marginal welfare value of a unit of aggregate consumption (see Nordhaus (2014) for more details).

[31] Additional graphs concerning the social cost of carbon and the stochastic evolution of global temper-

		Cum. emissions in 2010-2100 ($GtCO_2$)	Additional optimal abatement	Social cost of carbon in 2015 ($\$/tCO_2$)	Temperature increase in 2100 (°C)	Average prob of catastrophe $\bar{p}(T^*)$	Distorted prob of catastrophe $\hat{p}(T^*)$
$\mu = 0$	$\rho = 0$	3301	13.4 %	20.4 $	2.91	6.6 %	6.6 %
	$\rho = 1$	3244	14.9 %	20.7 $	2.89	6.5 %	6.4 %
	$\rho = 4$	3045	20.1 %	21.8 $	2.82	6.2 %	5.7 %
	$\rho = 10$	2634	30.9 %	24.1 $	2.67	5.5 %	3.9 %
$\mu = 1$	$\rho = 0$	3290	13.7 %	20.5 $	2.91	6.5 %	6.6 %
	$\rho = 1$	3230	15.3 %	20.8 $	2.89	6.4 %	6.4 %
	$\rho = 4$	3022	20.7 %	21.9 $	2.81	6.1 %	5.7 %
	$\rho = 10$	2585	32.2 %	24.4 $	2.65	5.4 %	4.0 %
$\mu = 10$	$\rho = 0$	3179	16.6 %	21.1 $	2.87	6.4 %	7.3 %
	$\rho = 1$	3096	18.8 %	21.5 $	2.84	6.2 %	7.2 %
	$\rho = 4$	2799	26.6 %	23.2 $	2.73	5.8 %	6.5 %
	$\rho = 10$	2195	42.4 %	27.0 $	2.50	4.8 %	4.8 %
Standard optimal version of DICE		3813	0 %	17.7 $	3.1	0%	0%

Table 1: Global cumulative emissions for the period 2010-2100, social cost of carbon in 2015, temperature increases (with respect to preindustrial level), average and distorted probabilities of catastrophe obtained with the modified version of DICE under the possibility of AMOC collapse.

last columns of Table 1 present the average $\bar{p}(T^*)$ and distorted $\hat{p}(T^*)$ probabilities of the AMOC collapse that is ultimately admitted by the DM. These values are computed at the optimal temperature endogenously calculated by the model. As expected, these probabilities are decreasing in both μ and ρ since the temperature is decreasing in both parameters. We also remark that $\bar{p}(T^*) < \hat{p}(T^*)$ as long as $\mu > \rho$, so that the DM always overestimates the probability of catastrophe when his model uncertainty aversion is stronger than his risk aversion (and vice-versa). Overall, these results from the stochastic IAM confirm that model uncertainty plays an important role in quantitative terms when the convergence of agreement effect is important. Depending on the parametrization of preferences, the possibility of a catastrophe, risk and model uncertainty aversion lead to additional mitigation effort of the cumulative emissions in the baseline scenario in the range of 13 to 49%.

In order to analyze the robustness of our results, we perform an extensive sensitivity analysis with respect to the most relevant model parameters and specifications. In particular, we take into account a different timing of the catastrophic event, different values for the equilibrium climate sensitivity, different utility discount rates, and different values of the economic losses of the catastrophe. While the full set of results is available in the Supplemental Material provided online, a summary of the results can be found in Table 2. We focus on the effect of model uncertainty aversion, while maintaining an intermediate value of $\rho = 4$ for the parameter of risk aversion.

Overall, the results show that the qualitative effect of model uncertainty aversion is robust throughout the specifications considered. For a lower value of the economic loss from the catastrophe (10% of GDP), a very low value of the climate sensitivity, or a

ature can be found in the online Supplemental Material.

[32] For these cases, cumulative emissions and temperature are reported for/until the year 2075.

		Increased abatement for $\mu = 0 \to 10$	Cum. emissions ($\mu = 10$) during 2010-2100 ($GtCO_2$)	Social Cost of Carbon ($\mu = 10$) in 2015 ($\$/tCO_2$)	Temperature increase ($\mu = 10$) in 2100 (°C)
Impact L	$L = 10\,\%$	1.3 %	3442	19.6 $	2.96
	$L = 30\,\%$	19.8 %	1953	30.0 $	2.37
Climate sensitivity	$ECS = 1.5$	2.4 %	4820	8.5 $	1.96
	$ECS = 4.5$	9.6 %	1540	42.5 $	3.16
Discount rate	prstp= 0.001	10.3 %	922	83.0 $	1.86
	prstp= 0.03	3.3 %	4306	9.1 $	3.20
Time of resolution[32]	2075	8.5 %	2091	26.8 $	2.32
	2100	5.4 %	2410	23.2 $	2.41
	2125	3.5 %	2616	21.1 $	2.46
Standard version ($\rho = 4, \mu = 10$)		8.1 %	2799	23.2 $	2.73

Table 2: Sensitivity Analysis. Differences in relative abatement are given in percentage point comparing the model run with $\mu = 10$ to $\mu = 0$, keeping $\rho = 4$ and everything else constant. Cumulative emissions, social cost of carbon and temperature increases are reported for the high model uncertainty aversion case ($\mu = 10$).

comparable high utility discount rate of 3%, the effect of model uncertainty is attenuated but still leads to a notable increase in optimal abatement. If, on the other hand, the values are set to the other side of the spectrum, model uncertainty raises precautionary mitigation effort significantly, and more than proportionally. For example, an impact L of 30% as opposed to 20% raises the social cost of carbon by about 7\$/tCO2. Finally, regarding the timing of learning and the potential occurrence of the catastrophic event, we find that the increase in abatement is higher for earlier occurrences and that it diminishes over time.

5 Conclusion

This paper aims at understanding and quantifying the impact of model uncertainty aversion on optimal abatement decisions, for the policy relevant case of catastrophic climate change. This attempt stems from the recognition that, although it is now fully recognized that the presence of these uncertainties represents an essential datum of the climate change issue, the way they are treated and integrated in the models used to make predictions or to design public policies remains unsatisfactory. By evaluating the optimal strategy for responding to an *uncertain* threat, the model we present in this paper has the advantage of treating policy analysis as that of a robust risk management problem.

In particular, we consider situations in which the actions we take today (such as choosing the level of abatement) affect the probability of incurring a high damage event (of catastrophic nature) in the future. The selection of optimal policies in this sense is essentially an exercise in risk management. However, the particularity of this exercise is that it is carried out under partial ignorance: the decision maker we study admits he does not know the exact relationship between her action and the probability of catastrophe. Rather, what the decision maker has available to help her making a choice is a collection of *models* or expert's estimates of this relationship. In contrast with purely risky situations in which the probabilities are known, the situations we study are therefore deeply uncertain or ambiguous. The ambiguity results precisely from the combination of risk and model uncertainty, and the decision maker's attitude towards ambiguity naturally results from the composition of attitudes towards these two distinct sources of uncertainty.

We compare this robust decision making approach with the standard expected utility approach, and show that the latter is not capable of differentiating distinct attitudes towards different types of uncertainty: it implicitly treats a situation in which experts have different dogmatic beliefs exactly the same way as a situation of pure risk. Rather, if the policy maker is ambiguity averse in the sense that he is more sensitive to model uncertainty than to risk, we show that he will undertake more abatement effort if the combination of the ambiguity prudence effect and the convergence of agreement effect is positive. The former condition is directly related to a condition about the functions representing preferences, while the latter is a characteristic of the available expert elicitation or model data. The intuition behind this result is that the desirability of preventive efforts is measured not only by the reduction in the expected damages, but also by the value of the associated reduced uncertainties. A degree of model disagreement that is decreasing in abatement effort is asking for a policy limiting global warming to relatively lower levels as it gives a precautionary policy maker an extra incentive for a more stringent mitigation policy, in the spirit of the precautionary principle. Finally, in contributing to answering the need to integrate the treatment of deep uncertainties and of possible catastrophic events in integrated assessment models, we apply our insights to the DICE model, and show that robust precautionary climate policies require a significantly higher abatement level. While the risk-neutral consideration of a catastrophic risk leads to a comparably low increase in abatement effort, this increase is magnified for reasonable degrees of both risk and model uncertainty aversion.

Although the proposed framework allows us to generate a set of original insights, many limitations remain. For example, we abstracted from the possibility of learning. Although it is unclear how much we can actually learn about these extreme climatic outcomes and what the implications are of learning on optimal abatement (IPCC, 2014), several insightful applications emphasizing the role of learning in integrated assessment models with tipping elements have been recently proposed (Rudik, 2015; Lemoine and Traeger, 2014). Our framework also requires calibrating parameters for which few estimates exist, such as the model uncertainty aversion, potentially limiting its practical use. Nonetheless, we believe that the flexibility of the model uncertainty decision framework, the results about the importance of the structure of model uncertainty, and the simplicity of the proposed metric of model disagreement, are widely applicable and can be fruitfully extended to other policy objectives and data-generating processes such as additional tipping elements, climate engineering, technological change, or even non climate-related policy issues.

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Appendix

A Proofs

A.1 Proof of Lemma 1

The proof directly follows from Proposition 4 in Berger (2014a). The condition to observe a higher (resp. lower) abatement due to ambiguity aversion may be written as

$$\mathrm{E}_\theta\left[\phi'\Big(\mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))\Big)\frac{\partial \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))}{\partial a}\right] \geq (\leq)\phi'\Big(\phi^{-1}\Big(\mathrm{E}_\theta\phi\big(\mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))\big)\Big)\Big)\mathrm{E}_\theta\frac{\partial \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))}{\partial a}. \tag{A.1}$$

Analogously to the risk theory literature, it can moreover be shown that CAAA is equivalent to: $\phi'\big(\phi^{-1}\big(\mathrm{E}\phi(\tilde{U})\big)\big) = \mathrm{E}\phi'(\tilde{U})$, strict DAAA to $\phi'\big(\phi^{-1}\big(\mathrm{E}\phi(\tilde{U})\big)\big) < \mathrm{E}\phi'(\tilde{U})$, and strict IAAA to $\phi'\big(\phi^{-1}\big(\mathrm{E}\phi(\tilde{U})\big)\big) > \mathrm{E}\phi'(\tilde{U})$. By letting $A \equiv \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))$ and $B \equiv \frac{\partial \mathrm{E}v(\tilde{C}_2(a,\tilde{\theta}))}{\partial a}$, we can rewrite condition (A.1) as $\mathrm{Cov}_\theta\big(\phi'(A);B\big) \geq (\leq)0$, or $\mathrm{Cov}_\theta\big(A;B\big) \leq (\geq)0$, since ϕ' is decreasing under ambiguity aversion. In the case of strict DAAA, we can use the chain of inequalities

$$\mathrm{E}_\theta\Big[\phi'(A)B\Big] \geq \mathrm{E}_\theta\phi'(A)\mathrm{E}_\theta B > \phi'\Big(\phi^{-1}\Big(\mathrm{E}_\theta\phi(A)\Big)\Big)\mathrm{E}_\theta B, \tag{A.2}$$

so that the left-hand side (LHS) of (A.1) is greater than the right-hand (RHS) side if $\mathrm{Cov}_\theta\big(A;B\big) \leq 0$, while in the case of strict IAAA, we can use the chain of inequalities

$$\mathrm{E}_\theta\Big[\phi'(A)B\Big] \leq \mathrm{E}_\theta\phi'(A)\mathrm{E}_\theta B < \phi'\Big(\phi^{-1}\Big(\mathrm{E}_\theta\phi(A)\Big)\Big)\mathrm{E}_\theta B, \tag{A.3}$$

to show that the RHS of (A.1) is greater than the LHS if $\mathrm{Cov}_\theta\big(A;B\big) \geq 0$. From Kimball (1951), it follows that this covariance is negative (resp. positive) if A and B are anticomonotonic (resp. comonotonic). $\square$

A.2 Proof of Proposition 1

Considering the ambiguity aversion function $\phi(U) = (h \circ v^{-1})(U)$, where U represents the expected utility computed in the presence of risk (i.e. $U \equiv \mathrm{E}v(\tilde{x})$), it is easy to compute the index of absolute ambiguity aversion as follows:

$$-\frac{\phi''(U)}{\phi'(U)} = -\frac{v'h'' - h'v''}{(v')^3}\frac{v'}{h'} = \frac{1}{v'}\left[-\frac{h''}{h'} + \frac{v''}{v'}\right], \tag{A.4}$$

where in a slight abuse of notations, we let $h \equiv h(v^{-1}(U))$ and $v \equiv v(v^{-1}(U))$. As expected, this ratio is positive if model uncertainty aversion is higher than risk aversion. Analogously to risk theory literature, DAAA means that $-\frac{\phi'''(U)}{\phi''(U)} \geq -\frac{\phi''(U)}{\phi'(U)}$, which is the case if and only if

$$\frac{h'''}{h'} + 2\left(\frac{-v''}{v'}\right)^2 \geq \frac{v'''}{v'} + \left(\frac{-h''}{h'}\right)^2 + \left(\frac{-h''}{h'}\right)\left(\frac{-v''}{v'}\right). \quad \square \tag{A.5}$$

A.3 Proof of Proposition 2

The result is obtained by decomposing $\sigma^2(a) = \mathrm{E}_\theta[p(a,\theta)^2] - p(a,\bar{\theta})^2$, where $\bar{p}(a) \equiv \mathrm{E}_\theta p(a,\tilde{\theta})$, and deriving this expression with respect to a: $\frac{\partial\sigma^2(a)}{\partial a} = 2\mathrm{E}_\theta[p(a,\theta)p_a(a,\theta)] - 2p(a,\bar{\theta})p_a(a,\bar{\theta}) = 2\mathrm{Cov}_\theta\Big(p(a,\theta);p_a(a,\theta)\Big)$. $\square$

B Economic impact uncertainty

Impact (or socioeconomic) uncertainty results from our "imperfect understanding of the impacts of climate change on human societies and of how these societies will respond" (Heal and Millner, 2014). In the context of our abatement model, imagine there is a scientific consensus on the link between the probability of a catastrophic event and the temperature increase (or abatement levels) given by a particular probability function $p(a)$. Even in this far from realistic situation of limited scientific uncertainty, there would however still be room for model uncertainty to play a significant role because of the remaining uncertainty concerning the economic impacts of a climate catastrophe. What, for example, would be the economic loss associated with a sea level rise of one meter? Would it be possible to construct protection dikes to save the most vulnerable places, and if so, at what cost? Or, what would be the cost associated with relocation and reconstruction? All these costs correspond to what we have called the *economic loss* associated with the catastrophic event, and are far from being perfectly known.[33] Different experts or studies may disagree on the total impact of a possible catastrophe, and this disagreement among economic models may potentially affect the decision made by a policy maker.

In our simple optimal abatement problem under impact uncertainty about the economic impacts L_s, the second period expected utility for a given model P_θ is now written as

$$\mathrm{E}v(\tilde{C}_2(a,\theta)) = p(a)\sum_{s\in S}\pi_s(\theta)v(w_2 - L_s) + (1-p(a))v(w_2), \tag{B.1}$$

where $\pi_s(\theta)$ denotes the probability according to expert θ of the loss L_s. The expected marginal benefit of abatement can be obtained as

$$\frac{\partial \mathrm{E}v(\tilde{C}_2(a,\theta))}{\partial a} = -p_a(a)\Big[v(w_2) - \sum_{s\in S}\pi_s(\theta)v(w_2 - L_s)\Big]. \tag{B.2}$$

Given that the probability of catastrophe is assumed to be decreasing in abatement, it is clear that expressions (B.1) and (B.2) will always be anticomonotonic, which leads us to the following result:

Proposition 4. *In the optimal abatement problem under model uncertainty about impacts, an agent considering the smooth criterion and exhibiting CAAA or DAAA always chooses to abate more than an expected utility maximizer .*

Proof. The result directly follows from Lemma 1. □

On the contrary, if the DM exhibits IAAA, it is impossible to unambiguously sign the final effect of model uncertainty aversion since it will depend on which of the two effects (degree of model disagreement or ambiguity prudence) dominates.

[33]Stern (2007) for example estimates the total loss for a high climate change scenario with non-market impacts and the risk of a catastrophe to be between 2.9% and 35.2% of GDP per capita in 2200 (see Figure 6.5 in Stern (2007) for more details).